



CYPRUS MATHEMATICAL SOCIETY  
NATIONAL COMPETITION  
DECEMBER 2017

GYMNASIUM B'

Date: 02/12/2017

Time: 09:30 -12:30

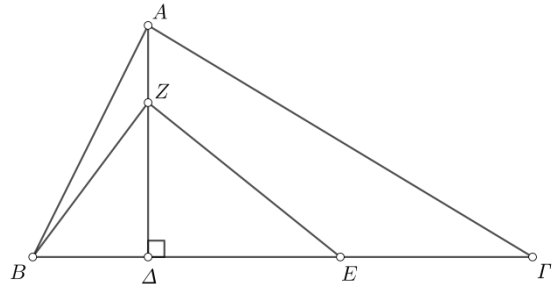
**INSTRUCTIONS**

1. Solve all the problems by giving full answers.
2. Each problem is marked with 10 points.
3. Write with blue or black ink (Shapes can be drawn with pencil).
4. The use of corrective liquid (Tip-Ex) is not allowed.
5. The use of a calculator is not allowed.

**PROBLEMS**

**Problem 1**

In the adjacent figure a triangle  $AB\Gamma$  is given.  $A\Delta$  is a height of the triangle  $AB\Gamma$ ,  $E$  is the midpoint of  $\Delta\Gamma$  and  $Z$  is a point on  $AD$ , such that the length of  $\Delta Z$  is twice the length of  $AZ$ . If the area of the triangle  $ABZ$  is  $5 \text{ cm}^2$  and the area of the quadrilateral  $A\Gamma EZ$  is  $30 \text{ cm}^2$ , calculate the area of the triangle  $BEZ$ .



**Problem 2**

Each of three friends,  $A, B$  and  $\Gamma$ , has a calculator and starts doing operations at the same time.  $A$  starts from number 100 and adds 3 at each step,  $B$  starts from number 2018 and subtracts 4 at each step, while  $\Gamma$  starts from number  $N$  and at the first step adds 1, at the second step 2, at the third step 3, etc. If after  $\nu$  steps the three friends end up with the same result, find number  $N$ .

**Problem 3**

George owes John €132. With how many different ways can George repay his debt, by using coins of €1 and banknotes of €5, €10;

*Note: In each way we are interested about the number of currencies and not about the order in which they are selected. For example, one way is "32 coins of €1 and 10 banknotes of €10".*

**Problem 4**

- (α) Show that  $\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta)$ , for all real numbers  $\alpha, \beta$ .
- (β) Let  $x, y$  be positive real numbers, such that  $x + y = xy = y^2 - x^2$ .
- i. Show that  $x - \frac{1}{x} = 1$ .
  - ii. Show that the number  $x^3 - \frac{1}{x^3}$  is an integer.