ΚΥΠΡΙΑΚΗ ΜΑΘΗΜΑΤΙΚΗ ΕΤΑΙΡΕΙΑ
ΠΑΓΚΥΠΡΙΟΣ ΔΙΑΓΩΝΙΣΜΟΣ

Α΄ ΛΥΚΕΙΟΥ

Ημερομηνία: *2/12/17* *Ώρα εξέτασης*: *09:30 -12:30*

Instructions:

1. Solve all the problems. Every problem has 10 points.
2. Write with blue or black ink (you can use pencil for the figures)
3. Use of correction fluid is not allowed.
4. Use of calculators is not allowed.

Problem1: Simplify the expression $A = \sqrt[5]{2\sqrt{5} - 3\sqrt{2}} \cdot \sqrt[10]{\frac{19+6\sqrt{10}}{2}}$

Problem 2: Let ν be a positive integer. Prove that:
(i) The sum Σ of the even integers, that exist between the positive integers $\nu^2 - \nu + 1$ and $\nu^2 + \nu + 1$, equals $\Sigma = \nu^3 + \nu$.
(ii) Integer $\Sigma + \nu$ is divisible by 3.

Problem 3: We consider two circles (C_1) and (C_2), that tangent each other externally at B and we draw their diameters AB and $B\Gamma$, respectively. We also draw the circle (C), with diameter $A\Gamma$. Let I be any point on one of the semicircles, with diameter $B\Gamma$ of the circle (C_2). The line IB intersects (C_1) at Λ and (C) at the points N, M such that Λ lies between N and B. From the center O of (C) we draw the perpendicular on MN, which intersects (C) at T.
Prove that $\angle NT\Lambda = \angle MTI$.

Problem 4: Determine all the pairs of positive and prime integers (μ, ν), such that the number $\mu^2 + 7\mu\nu + 9\nu^2$ to be a perfect square of positive integer. (A positive integer A is a perfect square of a positive integer, if there exists a positive integer α, such that $A = \alpha^2$).