



CYPRUS MATHEMATICAL SOCIETY
A' SELECTION COMPETITION IMC (Key Stage III)
MARCH 2018

Date: 7/03/2018

Time: 15:45 -17:45

Instructions:

1. Solve all of the problems. In problems **1 and 2** you should **show your work fully**. In problems **3, 4, 5 and 6** you should **write only the answer**.
2. Write with blue or black ink. (You may use pencil for the figures)
3. Do not use corrector liquid (Tipp-ex).
4. Do not use calculators.

Solve problems 1 and 2, **showing your work fully**.
Each problem is **10 points**.

Problem 1

The positive real numbers x, y satisfy the equation $x^2 + 9y^2 = \frac{13}{2}xy$.

Determine the largest and smallest possible values of $A = \frac{x-3y}{x+3y}$.

Problem 2

Consider the square $AB\Gamma\Delta$. Let E be a point on the side $\Gamma\Delta$. The bisector of the angle BAE intersects $B\Gamma$ at Z . The perpendicular from Δ to AZ intersects AE , AZ and AB at H , Θ and I , respectively. Prove that:

- a) The triangle ΔEH is isosceles.
- b) $AE = BZ + \Delta E$

Solve problems 3, 4, 5 and 6 **giving only the final answer**.
Each problem is **5 points**.

Problem 3

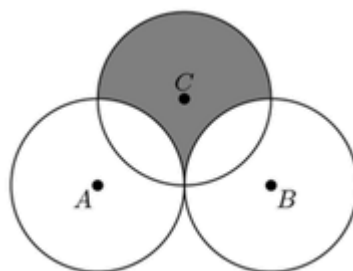
A bag contains 1-euro and 2-euro coins. Each 1-euro coin weighs 7.5 grams and each 2-euro coin weighs 8.5 grams. The 1-euro coins in the bag have the same total value as the 2-euro coins. If the total weight of the coins is 1175 grams, find the number of 1-euro coins, and the number of 2-euro coins in the bag.

Problem 4

A four-digit number $X: \overline{\alpha\beta\gamma\delta}$ has distinct digits with no digit equal to zero. We form the number $Y: \overline{\delta\gamma\beta\alpha}$ by reversing the order of the digits of X . If the sum of X and Y is 14773, determine the largest possible value of the smallest of the two numbers.

Problem 5

Each of the circles A, B and C has radius 1. Circles A and B are tangent. Circle C is tangent on the line segment AB , with point of tangency the midpoint of AB . What is the area inside circle C , but outside circle A and circle B ?



Problem 6

How many rectangles are there in a 6x6 chessboard that contain neither the cell (2,5) nor the cell (4,3)? (The rows and columns are indexed by 1,2,...,6.)

