CYPRUS MATHEMATICAL SOCIETY
REGIONAL COMPETITION
NOVEMBER 2017

LYCEUM C'

Date: 11/11/2017 *Time: 10:00 -12:00*

INSTRUCTIONS

1. Solve all the problems by giving full answers.
2. Each problem is marked with 10 points.
3. Write with blue or black ink (Shapes can be drawn with pencil).
4. The use of corrective liquid (Tip-Ex) is not allowed.
5. The use of a calculator is not allowed.

PROBLEMS

Problem 1 : (a) We consider the sequence $\alpha_0, \alpha_1, \alpha_2, \dots, \alpha_\nu, \dots$ with $\alpha_0 = 0$, $\alpha_1 = 1$ and

$$\alpha_{\nu+1} - 2\alpha_\nu + \alpha_{\nu-1} = 2\,, \qquad \forall \nu \in \{1, 2, 3, \dots\}.$$

Prove that: $\alpha_\nu = \nu^2$, $\forall \nu \in \{0, 1, 2, 3, \dots\}$.

(b) Prove that $\alpha_{\nu+1} + \alpha_\nu - 1$ is divisible by 4, for any $\nu \in \{0, 1, 2, 3, \dots\}$.

Problem 2 : On an orthogonal system axes we draw an equilateral triangle $\varDelta OAB$ with $B(6, 0)$, where O the origin and the point A lies on the first quadrant. From a random point $P(2\rho, 0)$ of the side OB $(0 < \rho < 3)$ we draw the parallel line to BA, which intersects the side OA at the point M. If $\varDelta$ is the intersection point of the medians of the triangle $\varDelta OMP$ and E is the midpoint of the segment AP, calculate the angles of the triangle $\varDelta B\varDelta E$.

Problem 3 : Let $f: [\alpha, \beta] \to \mathbb{R}$ be a function, with the properties:

- Continuous on $[\alpha, \beta]$
- $f''(x) > 0$, $\forall x \in (\alpha, \beta)$
- $f(\alpha) < 0 < f(\beta)$.

Prove that there is a unique $\rho \in (\alpha, \beta)$ such that $f(\rho) = 0$.

Problem 4 : The medians $B\varDelta, \varGamma E$ of a triangle $\varDelta AB\varGamma$ intersect at the point $\varTheta$. Prove that:
The quadrilateral $AE\varTheta\varDelta$ is circumscribed if and only if $AB = A\varGamma$.