



CYPRUS MATHEMATICAL SOCIETY
REGIONAL COMPETITION
NOVEMBER 2017

LYCEUM B'

Date: 11/11/2017

Time: 10:00 -12:00

INSTRUCTIONS

1. Solve all the problems by giving full answers.
2. Each problem is marked with 10 points.
3. Write with blue or black ink (Shapes can be drawn with pencil).
4. The use of corrective liquid (Tip-Ex) is not allowed.
5. The use of a calculator is not allowed.

PROBLEMS

Problem 1 : If $\cos x - \sin x = \sqrt{2}\sin x$ holds, prove that $\cos x + \sin x = \sqrt{2}\cos x$

Problem 2 : (a) Prove that the real roots of the equation

$$x^3 - 2x + 1 = 0$$

are $x_1 = 1$, $x_2 = \frac{-1-\sqrt{5}}{2}$, $x_3 = \frac{-1+\sqrt{5}}{2}$.

(b) Given that: $(x^3 + 1)^3 = 8(2x - 1)$, $x \in \mathbb{R}$ (1) and $y = \sqrt[3]{2x - 1}$ $y \in \mathbb{R}$ (2).

(i) Prove that: $x^3 - y^3 = 2(y - x)$.

(ii) Find all $x \in \mathbb{R}$, which satisfy the equation (1).

Problem 3 : On an orthogonal system axes we draw an equilateral triangle $\triangle OAB$ with $B(6, 0)$, where O the origin and the point A lies on the first quadrant. From a random point $P(2\rho, 0)$ of the side OB ($0 < \rho < 3$) we draw the parallel line to BA , which intersects the side OA at the point M . If Δ is the intersection point of the medians of the triangle $\triangle OMP$ and E is the midpoint of the segment AP , calculate the angles of the triangle $\triangle B\Delta E$.

Problem 4 : Given a triangle $\triangle AB\Gamma$ with sides $AB = \gamma$, $B\Gamma = \alpha$, $\Gamma A = \beta$ and $\alpha < \beta < \gamma$.

(i) Prove that $\angle A < 60^\circ$.

(ii) If μ is the length of the median $A\Delta$, prove that $4\mu^2 = \beta^2 + \gamma^2 + 2\beta\gamma\cos A$.

(iii) Prove that $\mu^2 > \frac{3\beta\gamma}{4}$.