



CYPRUS MATHEMATICAL SOCIETY
REGIONAL COMPETITION
NOVEMBER 2017

LYCEUM A'

Date: 11/11/2017

Time: 10:00 -12:00

INSTRUCTIONS

1. Solve all the problems by giving full answers.
2. Each problem is marked with 10 points.
3. Write with blue or black ink (Shapes can be drawn with pencil).
4. The use of corrective liquid (Tip-Ex) is not allowed.
5. The use of a calculator is not allowed.

PROBLEMS

Problem 1 : (a) Simplify the expression

$$A = \frac{(v^5 - 10v^3 + 9v)(v^2 - 4)}{v^2 + 3v}$$

(b) Prove that A is divisible by 12 for any natural number v .

Problem 2 : If $\cos x - \sin x = \sqrt{2}\sin x$ holds, prove that :

(a) $\tan x = \sqrt{2} - 1$

(b) $\cos x + \sin x = \sqrt{2}\cos x$

Problem 3 : (a) Prove that the real roots of the equation

$$x^3 - 2x + 1 = 0$$

are $x_1 = 1$, $x_2 = \frac{-1-\sqrt{5}}{2}$, $x_3 = \frac{-1+\sqrt{5}}{2}$.

(b) Given that: $(x^3 + 1)^3 = 8(2x - 1)$, $x \in \mathbb{R}$ (1) and $y = \sqrt[3]{2x - 1}$ $y \in \mathbb{R}$ (2).

(i) Prove that: $x^3 - y^3 = 2(y - x)$.

(ii) Find all $x \in \mathbb{R}$ which satisfy the equation (1).

Problem 4 : Given a square $AB\Gamma\Delta$. Let M be a point on the side $B\Gamma$ of the square. The bisector of the angle $\angle\Delta AM$ intersects the side $\Delta\Gamma$ at the point N . We draw the perpendicular line from N to the line AM , which intersects the line AB at the point H . Prove that $AH = BM + \Delta N$.